From computation to a reconstruction of (linear) logic

Team LoVe – LIPN Université Sorbone Paris Nord

Boris ENG (advisor: Thomas Seiller)

Context

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Traditional proof theory logic ⟶ mathematical tools

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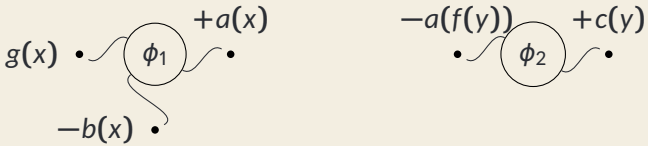
- ↳ Assumption : a reconstruction of logic starts from linear logic.
- ↳ Goal : make the logical mechanisms explicit.

Stellar Resolution

The space of computation

Independent stars with (un)polarised first-order term as rays.
Constellations (kind of programs) as multisets of stars.



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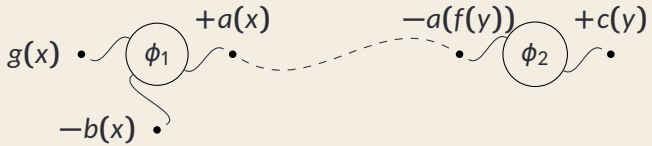

t and u are matchable with unifier $\theta = \{x \mapsto f(y)\}$.

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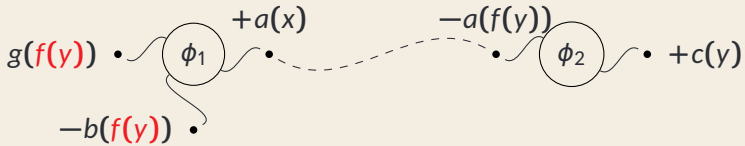

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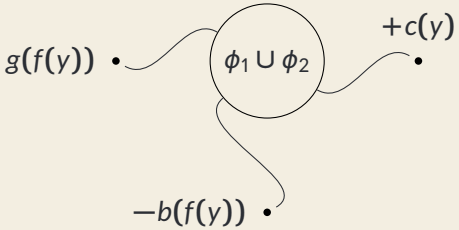

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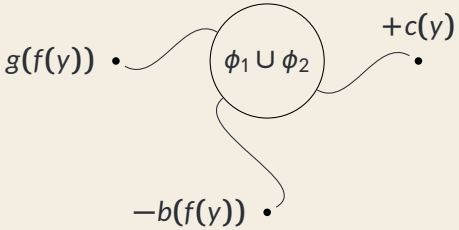

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Accidentally : (query-free) logic programming and tiling meet (e.g DNA computing).

What is a proof ?

From proof trees to proof structures

Proof tree π :

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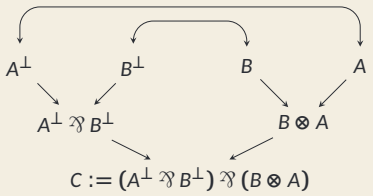

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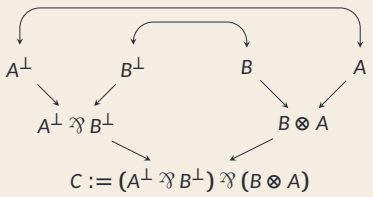

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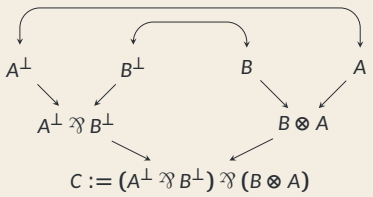

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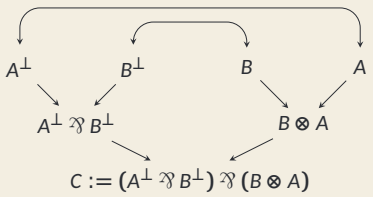

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↳ typing by stereotypes : passing $T_1, \ldots, T_n$ implies $\Phi : C$.

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Adequation $\Phi \in \mathbf{A}$ behaves as expected from the tests for **A**.

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Thank you for listening !